

In situ measurement of the dynamic structure factor in ultracold quantum gases

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We propose an experimental setup to efficiently measure the dynamic structure factor of ultracold quantum gases. Our method uses the interaction of the trapped atomic system with two different cavity modes, which are driven by external laser fields. By measuring the output fields of the cavity the dynamic structure factor of the atomic system can be determined. Contrary to previous approaches the atomic system is not destroyed during the measurement process.

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I. INTRODUCTION

The rapid experimental progress in the manipulation of ultracold quantum gases has enabled the creation of strongly correlated many-body systems, which are challenging to describe theoretically [1]. The unambiguous identification of these phases requires precise measurements to characterize its properties. A powerful tool represents the response function, which provides static properties of the system as well as reveals information about the excitations of the examined state. Here, we propose a method for an *in situ* measurement of the response function to a external and weak probing field for a trapped gas of ultracold atoms.

In seminal experimental measurements the Bogoliubov excitation spectrum in a Bose-Einstein condensate has been measured by probing the dynamic structure factor [2, 3]. Nowadays, this method has been applied to access the excitation spectrum within the BEC-BCS crossover [4, 5], as well as the smooth transition of the excitation gap from the superfluid to the Mott insulating phase [6]. A major drawback of the present approaches is that the setup measures the number of excitations created in the systems: the atomic system is destroyed during the measurement process, and in addition, it requires the application of strong fields violating in most situations the condition of a weak probe. As a consequence, it becomes difficult to distinguish, whether the measurements probes the dynamical evolution of a strongly driven system or characterizes the ground state properties. Several alternative methods to access information on ground state properties by measuring the light field passing through an atomic system have been discussed, e.g., the detection of magnetic order [7–10] and particle fluctuations [11, 12]. Within a remarkable experiment, the possibility to access the dynamical structure factor by a measurement of the probe field has recently been demonstrated

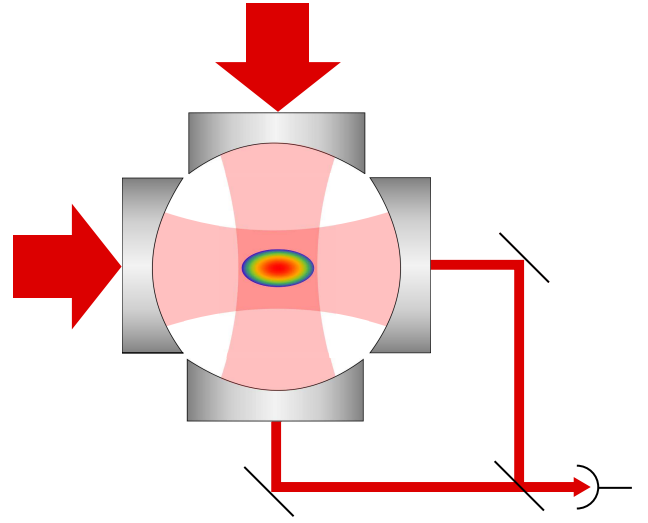


FIG. 1: Proposed experimental setup: an ultracold atomic cloud interacts with two different cavity fields, which are driven by external lasers. The light fields leaving the cavity contain information on the dynamic structure factor of the atomic system, which can be recovered in a homodyne detection scheme.

[13], which opens a way for weakly probing and analyzing cold atomic gases.

In this article, we propose a setup for realizing an *in situ* measurement of the linear response function for a cloud of trapped ultracold atomic atoms. The setup uses two different cavity fields, see Fig 1, and is motivated by recent progress of Bose-Einstein condensates coupled to high finesse cavities [14–17]. Note that the experiment can also be performed in a ring cavity setup to avoid standing waves created by the probe lasers; similar setups have been proposed for cavity state preparation [18, 19]. The main idea is that the interaction of the atomic system with the cavity modes transfers information on the response of the atomic system onto the cavity fields [8, 11]. By measuring the fields that leave the cavities one can reconstruct the linear response function as well as the dynamic structure factor. Our method puts

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only modest requirements on the cavity finesse and allows for fast measurements with sufficiently high photon count rates without destroying the atomic system.

For simplicity, we focus in the following on the density response function; the extension to alternative setups accessing e.g., the spin structure factor is straight forward. For weak probe fields ϕ , the linear response function is defined by the deformation of the particle density

$$\langle \rho_{\mathbf{q}}(\omega) \rangle = \chi_{\mathbf{q}}(\omega) \phi_{\mathbf{q}}(\omega) \quad (1)$$

with the Fourier transformation relation $\phi_{\mathbf{q}}(\omega) = \int dt d\mathbf{r} \phi(t, \mathbf{r}) \exp(i\omega t - i\mathbf{q}\mathbf{r})$. While the real part of the response function χ' accounts for the dispersive properties of the media on the probe field, the imaginary part χ'' describes the creation of excitations within the media. Consequently, the response function contains important information about the two-particles excitations as well as collective excitations, which is obvious from its relation to the dynamical structure factor

$$\chi''_{\mathbf{q}}(\omega) = -\pi[1 - \exp(-\beta\hbar\omega)]S(\mathbf{q}, \omega). \quad (2)$$

In previous experiments the dynamic structure factor $S(\mathbf{q}, \omega)$ has been studied extensively. However, it is important to point out, that the real part and the imaginary part of the response function are not independent of each other, but rather are related via the Kramers-Kronig relation

$$\chi''_{\mathbf{q}}(\omega) = -\frac{2}{\pi}\omega \int_0^{\infty} d\omega' \frac{\chi'_{\mathbf{q}}(\omega')}{\omega'^2 - \omega^2}. \quad (3)$$

As a consequence, it is possible to access information about the excitation spectrum by probing the real part of the response function off-resonantly and without creating any excitations in the media. Such a method is in strong contrast to the current approach, where the created excitations strongly distort the probed state. The method present in this manuscript allows one to access both the real part of the response function as well as the imaginary part, and opens a way to study *in situ* and for very weak probes the response of a quantum many-body system with cold atomic and molecular gases.

II. HAMILTONIAN DESCRIPTION

We consider a system of N atoms with two internal states $|g\rangle$ and $|e\rangle$ (see Fig. 2), coupled to two cavity fields $a_{\mathbf{k}}$ and $a_{\mathbf{p}}$ with frequencies $\omega_{\mathbf{k}}$ and $\omega_{\mathbf{p}}$, respectively. The dynamics of the cavity modes is governed by the Hamiltonian

$$H_C = \hbar\omega_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}} + \hbar\omega_{\mathbf{p}}a_{\mathbf{p}}^{\dagger}a_{\mathbf{p}}. \quad (4)$$

For a large detuning Δ_e of the cavity fields from the atomic resonance, we can adiabatically eliminate the state $|e\rangle$. Ignoring effects from spontaneous emission, the

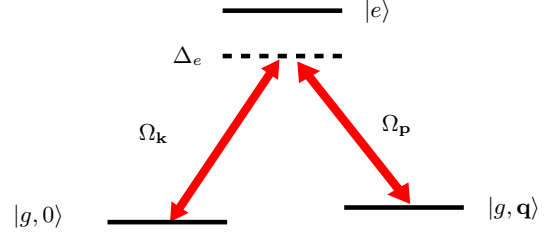


FIG. 2: Internal level structure of the atomic system. Transitions between the ground state $|g\rangle$ and the excited state $|e\rangle$ are driven by cavity fields $\Omega_{\mathbf{k}}$, $\Omega_{\mathbf{p}}$ with a large detuning Δ_e from the atomic resonance. The momentum difference $\mathbf{q} = \mathbf{k} - \mathbf{p}$ of the cavity fields is transferred to the atomic system.

interaction between the two cavity fields and the particle density operator $\rho_{\mathbf{q}}$ of the atomic system takes the form

$$H_I = g \left[\rho_0 \left(a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + a_{\mathbf{p}}^{\dagger} a_{\mathbf{p}} \right) + \rho_{-\mathbf{q}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{p}} + \rho_{\mathbf{q}} a_{\mathbf{p}}^{\dagger} a_{\mathbf{k}} \right], \quad (5)$$

with g being the two-photon coupling constant, while $\mathbf{q} = \mathbf{k} - \mathbf{p}$ denotes the difference in momenta between the cavity fields. The first term in H_I accounts for the Stark shift of the atoms and only leads to a renormalization of the cavity resonance frequencies, i.e., $\bar{\omega}_{\mathbf{k}, \mathbf{p}} = \omega_{\mathbf{k}, \mathbf{p}} + \rho_0 g / \hbar$. Note that here we assume ρ_0 denotes the total number of atoms in the cavity and is a conserved quantity. In addition we would also like to point out that the cavity lasers can also be focused and probe only parts of the atomic system. Then, ρ_0 accounts only for the atoms within the mode volume of the lasers, and the Stark shift becomes a local shift in the chemical potential.

III. WEAK COUPLING EXPANSION

A. Free field solution

We consider the terms involving the density operator $\rho_{\pm\mathbf{q}}$ as small perturbations and solve the system first in absence of these coupling terms. Then, the quantum Langevin equation for the cavity fields reduce to

$$\dot{a}_{\mathbf{k}} = -i\bar{\omega}_{\mathbf{k}}a_{\mathbf{k}} - \frac{\gamma}{2}a_{\mathbf{k}} + \sqrt{\gamma}b_{\mathbf{k}}^{\text{in}}(t) \quad (6)$$

$$\dot{a}_{\mathbf{p}} = -i\bar{\omega}_{\mathbf{p}}a_{\mathbf{p}} - \frac{\gamma}{2}a_{\mathbf{p}} + \sqrt{\gamma}b_{\mathbf{p}}^{\text{in}}(t), \quad (7)$$

where $b_{\mathbf{k}, \mathbf{p}}^{\text{in}}(t)$ are the input fields of the cavities [20]. We assume that the cavity is driven by two lasers with frequencies $\Omega_{\mathbf{k}}$ and $\Omega_{\mathbf{p}}$, i.e., $\langle b_{\mathbf{k}, \mathbf{p}}^{\text{in}}(t) \rangle = \beta_{\mathbf{k}, \mathbf{p}} \exp(-i\Omega_{\mathbf{k}, \mathbf{p}}t)$. Then, the steady state solution for the averaged fields $\alpha_{\mathbf{k}, \mathbf{p}} = \langle a_{\mathbf{k}, \mathbf{p}} \rangle$ is given by

$$\alpha_{\mathbf{k}}(t) = \frac{\sqrt{\gamma}}{\gamma/2 + i(\bar{\omega}_{\mathbf{k}} - \Omega_{\mathbf{k}})} \beta_{\mathbf{k}} \exp(-i\Omega_{\mathbf{k}}t) \quad (8)$$

$$\alpha_{\mathbf{p}}(t) = \frac{\sqrt{\gamma}}{\gamma/2 + i(\bar{\omega}_{\mathbf{p}} - \Omega_{\mathbf{p}})} \beta_{\mathbf{p}} \exp(-i\Omega_{\mathbf{p}}t). \quad (9)$$

To simplify the analysis, we set the cavity detunings to zero, $\Delta = \bar{\omega}_{\mathbf{k}} - \Omega_{\mathbf{k}} = \bar{\omega}_{\mathbf{p}} - \Omega_{\mathbf{p}} = 0$ and assume equal driving fields $|\beta_{\mathbf{k}}| = |\beta_{\mathbf{p}}|$, while the generalization to unequal driving fields is straightforward. Furthermore, we can absorb a phase factor in the definition of $\beta_{\mathbf{k},\mathbf{p}}$ such that $\alpha_{\mathbf{k}}(t) = \alpha_0 \exp(-i\Omega_{\mathbf{k}}t)$ and $\alpha_{\mathbf{p}}(t) = \alpha_0 \exp(-i\Omega_{\mathbf{p}}t)$ with $\alpha_0 = 2|\beta_{\mathbf{k}}|/\sqrt{\gamma}$.

B. Effects of fluctuations

To study the effect of the atom-field interaction, we consider small fluctuations around the free field solution. For this, we define a new set of operators as

$$A = \frac{1}{\sqrt{2}} (a_{\mathbf{k}} e^{i\Omega_{\mathbf{k}}t} + a_{\mathbf{p}} e^{i\Omega_{\mathbf{p}}t}) \quad (10)$$

$$B = \frac{1}{\sqrt{2}} (a_{\mathbf{k}} e^{i\Omega_{\mathbf{k}}t} - a_{\mathbf{p}} e^{i\Omega_{\mathbf{p}}t}) \quad (11)$$

with the inverse transformation

$$a_{\mathbf{k}} = \frac{1}{\sqrt{2}} (A + B) e^{-i\Omega_{\mathbf{k}}t} \quad (12)$$

$$a_{\mathbf{p}} = \frac{1}{\sqrt{2}} (A - B) e^{-i\Omega_{\mathbf{p}}t}. \quad (13)$$

These operators have the expectation values $\langle A \rangle = 2\sqrt{2/\gamma}\beta_{\mathbf{k}}$ and $\langle B \rangle = 0$, and we expand them into their mean values and small fluctuations $\delta A, \delta B$. Then, the interaction Hamiltonian $H_I = H_1 + H_2 + H_3$ reduces to

$$H_1 = \int d\mathbf{x} \rho(\mathbf{x}) V_0 \cos(\mathbf{q}\mathbf{x} - \omega t) \quad (14)$$

$$H_2 = \frac{g_{\text{eff}}}{2} (\rho_{\mathbf{q}} e^{-i\omega t} + \rho_{-\mathbf{q}} e^{i\omega t}) (\delta A + \delta A^\dagger) \quad (15)$$

$$H_3 = \frac{g_{\text{eff}}}{2} (\rho_{\mathbf{q}} e^{-i\omega t} - \rho_{-\mathbf{q}} e^{i\omega t}) (\delta B - \delta B^\dagger) \quad (16)$$

with $\omega = \Omega_{\mathbf{k}} - \Omega_{\mathbf{p}}$, the potential strength $V_0 = g|\langle A \rangle|^2$, and the effective coupling $g_{\text{eff}} = g|\langle A \rangle|$. The first term describes a classical driving field $V_{\text{ext}} = V_0 \cos(\mathbf{q}\mathbf{x} - \omega t)$ for the atomic system, while the last terms account for the coupling between the atomic system and the cavity fields. For $\omega = 0$, this set of equations reduces to the case previously studied in the context of cavity cooling [21]. We now introduce the quadrature operators

$$X_B = \frac{1}{2} [\delta B + \delta B^\dagger] \quad P_B = -i [\delta B - \delta B^\dagger] \quad (17)$$

$$X_A = \frac{1}{2} [\delta A + \delta A^\dagger] \quad P_A = -i [\delta A - \delta A^\dagger] \quad (18)$$

with the commutation relations

$$[X_A, P_A] = [X_B, P_B] = i \quad (19)$$

$$[X_B, P_A] = [X_A, P_B] = 0. \quad (20)$$

The equation of motions for the cavity field then reduce to

$$\dot{X}_A = -\frac{\gamma}{2} X_A \quad (21)$$

$$\dot{P}_A = -\frac{g_{\text{eff}}}{\hbar} [\rho_{\mathbf{q}} e^{-i\omega t} + \rho_{-\mathbf{q}} e^{i\omega t}] - \frac{\gamma}{2} P_A \quad (22)$$

$$\dot{X}_B = i\frac{g_{\text{eff}}}{\hbar} [\rho_{\mathbf{q}} e^{-i\omega t} - \rho_{-\mathbf{q}} e^{i\omega t}] - \frac{\gamma}{2} X_B \quad (23)$$

$$\dot{P}_B = -\frac{\gamma}{2} P_B. \quad (24)$$

These equations describe the back action of the atomic system with the particle density $\rho_{\mathbf{q}}$ onto the cavity fields, i.e., a fluctuation in the particle density operator of the atomic system $\rho_{\mathbf{q}}(t)$ influences the cavity fields within characteristic quantum-non-demolition setup.

C. Linear response regime

The leading term H_1 drives a perturbation onto density of the atomic system with the field $\phi(t, \mathbf{x}) = V_0 \cos(\mathbf{q}\mathbf{x} - \omega t)$. The condition of a weak probe field reduces to $V_0 < E_s$ with $E_s \sim \hbar^2 \mathbf{q}^2 / m$ the characteristic energy scale of the atomic system [22]. Then, the response of the atomic system to this external probe is well described within linear response theory

$$\langle \rho(t, \mathbf{x}) \rangle = \int dt' d\mathbf{x}' \chi(t - t', \mathbf{x} - \mathbf{x}') \phi(t', \mathbf{x}'), \quad (25)$$

which implies for the present drive with momentum transfer \mathbf{q} and frequency ω the response $\langle \rho_{\mathbf{q}}(\omega) \rangle = V_0 \chi_{\mathbf{q}}(\omega)/2$. Then, it is possible to replace the operators in the Langevin equations by their expectation values, and we obtain the coupled differential equations

$$\frac{d}{dt} \langle X_A \rangle = -\frac{\gamma}{2} \langle X_A \rangle \quad (26)$$

$$\frac{d}{dt} \langle P_A \rangle = -\frac{g_{\text{eff}}}{\hbar} V_0 \chi'_{\mathbf{q}}(\omega) - \frac{\gamma}{2} \langle P_A \rangle \quad (27)$$

$$\frac{d}{dt} \langle X_B \rangle = -\frac{g_{\text{eff}}}{2\hbar} V_0 \chi''_{\mathbf{q}}(\omega) - \frac{\gamma}{2} \langle X_B \rangle \quad (28)$$

$$\frac{d}{dt} \langle P_B \rangle = -\frac{\gamma}{2} \langle P_B \rangle. \quad (29)$$

We can immediately identify the steady state solution

$$\langle X_A \rangle = \langle P_B \rangle = 0 \quad (30)$$

$$\langle P_A \rangle = -\frac{2g_{\text{eff}}V_0}{\hbar\gamma} \chi'_{\mathbf{q}}(\omega) \quad (31)$$

$$\langle X_B \rangle = -\frac{g_{\text{eff}}V_0}{\hbar\gamma} \chi''_{\mathbf{q}}(\omega). \quad (32)$$

Consequently, the linear response of the atomic system is imprinted onto the cavity fields: the imaginary part of the response function describing the creation of two-particle excitations and collective excitations in the atomic system is encoded by a shift in the amplitude operator X_B accounting for the scattered of photons from

one cavity mode onto the other. In turn, the real part of the response function characterizing the dispersive part of the media leads to a shift in the phase P_A of the fields.

IV. EXPERIMENTAL PARAMETERS

Finally, we provide the experimental parameters for the measurement of the response function. We will focus on the measurement of the phase quadrature $\langle P_A \rangle$, as it exhibits two advantages over the measurement of $\langle X_B \rangle$: First, we note that $\langle P_A \rangle$ is directly given by the sum of the expectation values $\langle P_{\mathbf{k}} \rangle$ and $\langle P_{\mathbf{p}} \rangle$ of the physical cavity fields $a_{\mathbf{k}}$ and $a_{\mathbf{p}}$, which can be efficiently measured within a homodyne detection. For appropriate phases of the cavity fields this means that the measurements are done against zero background, which is not the case when measuring $\langle X_B \rangle$. Second, the Kramers-Kronig relation Eq. (3) allows us to compute $\chi_q''(\omega)$ *on resonance* by using measurement data taken *off resonance*. Therefore, we are able to obtain the dynamic structure factor of the atomic system without transferring energy onto the atomic system.

The validity of the expansion and ignoring the quadratic terms in the fluctuating fields requires a large photon number in the cavity, i.e., $\langle A \rangle \gg 1$. In turn, the weak driving field $V_0 \lesssim E_s$ implies a weak coupling $g = V_0/|\langle A \rangle|^2 \lesssim E_s/|\langle A \rangle|^2$. Such weak coupling is easily achieved by controlling the detuning to the excited level, see Fig. 2. Note, that this weak coupling is in high contrast to the recent experimental works on high finesse cavities [14–17], and significantly simplifies the requirements for the experimental realization of such a setup. The phase quadratures can be efficiently been measured within a homodyne detection with the sensitivity, see Fig. 3,

$$S = \frac{\langle P_A \rangle}{\langle A \rangle} = \frac{\langle P_{\mathbf{k}} \rangle + \langle P_{\mathbf{p}} \rangle}{\langle A \rangle}. \quad (33)$$

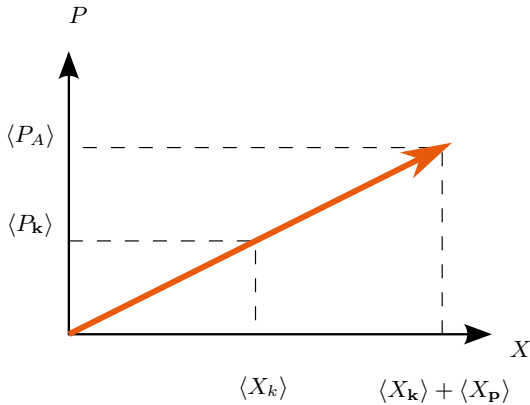


FIG. 3: Phasor diagram illustrating the measurement of $\langle P_A \rangle$ as the sum of the physical observables $\langle P_{\mathbf{k}} \rangle$, $\langle P_{\mathbf{p}} \rangle$. A signal can be detected only when the ratio $\langle P_A \rangle / (\langle X_{\mathbf{k}} \rangle + \langle X_{\mathbf{p}} \rangle)$ is large compared to phase fluctuations.

The range of interesting values for the response function are $\chi_q(\omega) \approx N/E_s$ with N the total number of particles and E_s the characteristic energy scale of the atomic system. This requires a sensitivity to measure the phase angle

$$S \approx \frac{Ng V_0}{\hbar\gamma E_s} = \frac{V_0}{\hbar\gamma} \frac{N}{|\langle A \rangle|^2} \frac{V_0}{E_s} \quad (34)$$

For the last equation, we have used the relation between the driving field $V_0 = g|\langle A \rangle|^2$ and the number of photons in the cavity. The number of atoms within the cavity are typically in the range $N \sim 10^6$, and therefore, with a cavity decay rate $\gamma \sim 100$ GHz corresponding to a quality factor $Q \sim 10^4$ with up to 100 photons within the cavity, and a characteristic energy $E_s \sim 1$ KHz, the required sensitivity is in the range $S \sim 10^{-4}$, which can be achieved with present techniques [23].

V. CONCLUSIONS

In summary we have demonstrated the experimental feasibility of an *in situ* measurement of the dynamic structure factor of arbitrary atomic systems, providing a novel tool for the coherent manipulation of ultracold quantum gases. The proposed method employs two cavity fields which are driven by external lasers. We have shown that the required resolution can be achieved using a cavity finesse that has already been reached in present experiments. Furthermore, the method allows for sufficiently fast measurements.

The current setup is in close analogy to the recently experimentally realized system [13]. The main difference is, that in the experimental setup a strong imbalance between the two photon modes is applied. As a consequence, the validity of the expansion up to linear order in the fluctuating fields is satisfied by the strong coupling laser, while for the weak probe beam a very low photon number can be used. Then, the present calculations can be performed again in a straightforward manner with the only modification, that the real part of the response function is written on the phase quadrature $P_{\mathbf{p}}$, while the imaginary part is encoded onto the amplitude quadrature $X_{\mathbf{p}}$. In the experiment the amplitude quadrature was accessed with a heterodyne detection [13], while the real part of the response function again could be probed by a simpler homodyne detection. We would like to point out that the contrast of the signal can be strongly enhanced by using a cavity for the weak probe field.

Finally, we would like to point out, that the present setup can also be used to study the relaxation of excitations in the atomic system: first, the lasers imprint several excitations on resonance into the atomic system, while at a later stage the probe of the real part allows one to analyze whether these excitations are still present or have relaxed.

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- [1] I. Bloch, J. Dalibard, and W. Zwerger, *Rev. Mod. Phys.* **80**, 885 (2008).
 - [2] D. M. Stamper-Kurn, A. P. Chikkatur, A. Görlitz, S. Inouye, S. Gupta, D. E. Pritchard, and W. Ketterle, *Phys. Rev. Lett.* **83**, 2876 (1999).
 - [3] J. Steinhauer, R. Ozeri, N. Katz, and N. Davidson, *Phys. Rev. Lett.* **88**, 120407 (2002).
 - [4] C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. H. Denschlag, and R. Grimm, *Science* **305**, 1128 (2004).
 - [5] J. T. Stewart, J. P. Gaebler, and D. S. Jin, *Nature* **454**, 744 (2008).
 - [6] T. Stöferle, H. Moritz, C. Schori, M. Köhl, and T. Esslinger, *Phys. Rev. Lett.* **92**, 130403 (2004).
 - [7] K. Eckert, L. Zawitkowski, A. Sanpera, M. Lewenstein, and E. S. Polzik, *Phys. Rev. Lett.* **98**, 100404 (2007).
 - [8] K. Eckert, O. Romero-Isart, M. Rodriguez, M. Lewenstein, E. S. Polzik, and A. Sanpera, *Nature Phys.* **4**, 50 (2008).
 - [9] I. de Vega, J. I. Cirac, and D. Porras, *Phys. Rev. A* **77**, 051804 (2008).
 - [10] J. S. Douglas and K. Burnett, *Phys. Rev. A* **82**, 033434 (2010).
 - [11] I. B. Mekhov, C. Maschler, and H. Ritsch, *Nature Phys.* **3**, 319 (2007).
 - [12] I. B. Mekhov and H. Ritsch, *Phys. Rev. Lett.* **102**, 020403 (2009).
 - [13] J. M. Pino, R. J. Wild, P. Makotyn, D. S. Jin, and E. A. Cornell, *Phys. Rev. A* **83**, 033615 (2011).
 - [14] S. Slama, S. Bux, G. Krenz, C. Zimmermann, and P. W. Courteille, *Phys. Rev. Lett.* **98**, 053603 (2007).
 - [15] F. Brennecke, T. Donner, S. Ritter, T. Bourdel, M. Köhl, and T. Esslinger, *Nature* **450**, 268 (2007).
 - [16] Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, *Nature* **450**, 272 (2007).
 - [17] K. W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, *Nature Phys.* **4**, 561 (2008).
 - [18] F. Mattinson, M. Kira, and S. Stenholm, *J. Mod. Opt.* **48**, 889 (2001).
 - [19] J. Larson and E. Andersson, *Phys. Rev. A* **71**, 053814 (2005).
 - [20] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 1994).
 - [21] A. Griessner, D. Jaksch, and P. Zoller, *J. Phys. B* **37**, 1419 (2004).
 - [22] L. Van Hove, *Phys. Rev.* **95**, 249 (1954).
 - [23] H. Hansen, T. Aichele, C. Hettich, P. Lodahl, A. I. Lvovsky, J. Mlynek, and S. Schiller, *Opt. Lett.* **26**, 1714 (2001).